Top quark contribution to hadronic decays of the Z-boson at α_s^2 in QCD

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Abstract

We evaluate the effect of a virtual top quark on the coefficient of α_s^2 in the decay rates $\Gamma(Z \to \text{hadrons})$ and $\Gamma(Z \to b\overline{b})$. We treat the dependence on the top quark mass exactly instead of using a large mass expansion. The present work completes the evaluation of the α_s^2 contributions to these quantities. The calculation uses both the $\overline{\text{MS}}$ and Collins-Wilczek-Zee renormalization prescriptions. The results can be applied to the hadronic decays of the τ-lepton.

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One crucial test of quantum chromodynamics (QCD) is based on the total cross section for $e^+e^- \rightarrow$ hadrons at LEP [1]. The theoretical expression for

$$R_Z = \Gamma(Z^0 \rightarrow \text{hadrons})/\Gamma(Z^0 \rightarrow e^+e^-) \tag{1}$$

is compared to the measured quantity to extract a value for $\alpha_s(M_Z)$. If QCD is correct, this value of $\alpha_s(M_Z)$ should be the same as the value obtained in other experiments, such as deeply inelastic scattering and three jet production in e^+e^- annihilation. A high precision experiment is required in order to obtain good precision on $\alpha_s(M_Z)$ because $\alpha_s(M_Z)$ is small and R_Z has the form

$$R_Z = R_0 \left\{ 1 + (\alpha_s/\pi) + \cdots \right\}. \tag{2}$$

Despite this experimental difficulty, the use of this measurement as a QCD test is regarded as "gold plated" because of the theoretical simplicity of R_Z. Simple analyticity arguments relate R_Z to the behavior of the Z-boson propagator at short distances, without the need for complicated arguments about factorization of long-distance effects, or about the infrared safety of jet definitions. To be sure, there are "hadronization effects," but at $M_Z \approx 90$ GeV, these are easily seen to be negligible.

The test may also be regarded as "gold plated" because calculations for R_Z are quite complete. The leading contributions in the approximation that the u, d, s, c, and b quark masses vanish while the t quark mass is large compared to M_Z are known up to order α_s^3 [2–5]. Corrections to this limit, including the complete dependence on m_t, are known for most graphs up to order α_s^2 [5,6]. The one exception concerns the α_s^2 graphs shown in Fig. 1, in which there is a top quark loop. Here, the result is known only in an expansion about $m_t/M_Z = \infty$ [7]. In the real world, it appears that m_t is not much greater than M_Z [8], so that it is not evident how good this approximation is. The purpose of this paper is to fill in this gap by calculating the graphs of Fig. 1 for arbitrary m_t/M_Z.

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FIG. 1. Contributions to the Z-boson propagator with a heavy quark loop (represented by the thick line). The weak current operators are represented by crosses. The discontinuity of these diagrams contributes to R_Z.

Our motivation for undertaking a calculation of what is, in fact, a small correction to R_Z was twofold. First, we wanted to know for sure that the correction is small. Second, we wanted to develop methods for calculating multiloop Feynman graphs with non-zero masses. There are powerful techniques that are used for multiloop massless graphs, but these techniques cannot be applied when masses are present. One can perform expansions in powers of m or $1/m$, as noted above, but when m is actually neither very large nor very small compared to the momentum scale of the problem, it would be better to be able to include m in the calculation without approximation. We anticipate applications of the techniques and results used in this paper to other problems, especially the calculation of the hadronic decay width of the τ-lepton. For the case of Z-boson decay, we have in mind that the heavy quark in Fig. 1 is the top quark, but, since the mass is arbitrary, the results may also be applied to bottom or charm quark loops.

In order to state clearly what contribution to R_Z we are calculating, we begin the exposition by outlining the theoretical structure of R_Z that is relevant for phenomenology. We define the perturbative expansion coefficients of R_Z by

$$R_Z = \sum_{n=0}^{\infty} \left(\frac{\alpha_s(N_L, \mu)}{\pi} \right)^n R_n \, . \tag{3}$$

Here α_s is evaluated at scale μ, as defined in the $\overline{\text{MS}}$ renormalization scheme with N_L light quark flavors. Here we may take a "light" flavor f to be one for which $m_f < \sqrt{s}/2$. For $\sqrt{s} \approx 90$ GeV, the number of light flavors is $N_L = 5$. At order α_s^0, the structure of R is simple:

$$R_0 = \sum_{f \leq N_L} \sum_{i=V,A} \mathcal{R}_f^i \left(m_f^2(\mu)/s \right) \, . \tag{4}$$

Here there is a sum over the light flavors f. There is also a sum over an index $i = (V, A)$ that labels the two contributions, vector, $\langle J_V^\mu J_V^\nu \rangle$, and axial vector, $\langle J_A^\mu J_A^\nu \rangle$. The corresponding contribution to R_Z at lowest order in the electroweak interactions and zeroth order in the strong interactions is denoted $\mathcal{R}_f^i$. These contributions are functions of m_f^2/s. For the sake of definiteness, we choose the $\overline{\text{MS}}$ definition of the quark mass $m_f(\mu)$. Then the mass depends on the renormalization scale μ. The values of the $\mathcal{R}_f^i$ may be found, for instance, in Ref. [9].

In the Born term (4) it is convenient to keep the exact dependence on the quark masses. For the QCD corrections, an expansion about $m_f^2/s = 0$ suffices since, in fact, $m_f^2/s \ll 1$ for the heaviest "light" quark, the b. Thus we write the first order corrections as

$$R_1 = \sum_{f \leq N_L} \sum_{i=V,A} \mathcal{R}_f^i(0) \left[1 + \left(\frac{m_f^2}{s} \right) \left(A_{1,0}^i + A_{1,1}^i \log \left(\mu^2/s \right) \right) \right] + \mathcal{O}(m_L^4/s^2). \tag{5}$$

Here $A_{1,0}^V = 12$, $A_{1,1}^V = 0$, $A_{1,0}^A = -22$, and $A_{1,1}^A = -12$. The value of $A_{1,1}^i$ follows simply from the renormalization group applied to the μ dependence of $m(\mu)$ in R_0; the $A_{1,0}^i$ are easily calculated and may be found in Ref. [6]. The notation $+\mathcal{O}(m_L^4/s^2)$ is intended to indicate that we have neglected terms that are no larger than m_L^4/s^2, where m_L denotes the heaviest of the light quark masses, times constants and logarithms of s, μ, and the quark masses.

At order α_s^2 we have

$$\begin{aligned}
R_2 = \sum_{f \leq N_L} \sum_{i=V,A} \mathcal{R}_f^i(0) & \left[1.985707 - 0.115295 N_L + \beta_0(N_L) \log \left(\mu^2/s \right) \right. \\
& + \left(\frac{m_f^2}{s} \right) \left(A_{2,0}^i + A_{2,1}^i \log \left(\mu^2/s \right) + A_{2,2}^i \log^2 \left(\mu^2/s \right) \right) \\
& \left. + \sum_{f' \leq N_L} F(m_{f'}^2/s) + \sum_{f' > N_L} G(m_{f'}^2/s) \right] \\
& + \mathcal{O}(m_L^4/s^2) + \mathcal{O}(m_L^2/s) \times \mathcal{O}(s/m_H^2) \\
& + \mathcal{L} \, .
\end{aligned} \tag{6}$$

Here the first line represents the result with N_L flavors of massless quarks, including all order α_s^2 graphs except that shown in Fig. 2. The presence of the term $\beta_0 \log(\mu^2/s)$, where

$\beta_0(N_L) = (33 - 2N_L)/12$, follows from the application of the renormalization group to the μ dependence of $\alpha_s(\mu)$ in R_1. The second line gives the first corrections due to the non-zero mass of the quark f to which the Z-boson couples. The values of $A_{2,1}^i$ and $A_{2,2}^i$ follow by direct calculation or by applying the renormalization group, while the $A_{2,0}^i$ follow from direct calculation and can also be found in Ref. [6]:

$$
\begin{aligned}
A_{2,0}^V &= 253/2 - 13N_L/3 & A_{2,0}^A &= -108.140 + 4.4852N_L \\
A_{2,1}^V &= 57 - 2N_L & A_{2,1}^A &= -155 + 16N_L/3 \\
A_{2,2}^V &= 0 & A_{2,2}^A &= -57/2 + N_L \,.
\end{aligned}
\tag{7}
$$

In the third line, we encounter the functions F and G that are the subject of this paper. Their presence is due to the graphs in Fig. 1. In these graphs, a quark of flavor f' appears in an internal loop. Each *light* quark f' (*i.e.* u, d, s, c, b for $\sqrt{s} \approx 90$ GeV) makes a contribution $-0.115295 + F(m_{f'}^2/s)$. Here, by definition, $F(0) = 0$. Each *heavy* quark (*i.e.* top) makes a contribution $G(m_{f'}^2/s)$. The decoupling theorem [10] guarantees that G vanishes as $m_{f'}^2/s \to \infty$. The first term in an expansion of G about $m_{f'}^2/s = \infty$ was given in Ref. [7]. Our object is to calculate G for $m_{f'}$ not much bigger that $\sqrt{s}$. The calculation of G yields a calculation of F as a corollary. In calculating F and G, we take the mass m_f of the quark in the outside loop to be zero. Thus in Eq. (6) we neglect terms from graphs with light quarks only that are of order m_L^4/s^2 times logarithms. In graphs like those of in Fig. 1 with a light quark on the outside loop and a heavy quark on the inside loop, we neglect terms of order m_L^2/s times s/m_H^2 times logarithms, where m_H is the lightest heavy quark mass (*i.e.* m_t).

We have yet to mention the term $\mathcal{L}$ in Eq. (6), which corresponds to the graph shown in Fig. 2. In the other graphs contributing to Eq. (6), top quark contributions vanish if one takes the limit $m_t \to \infty$. This does not happen here because the top quark is required in order to cancel the contribution of the bottom quark to the triangle anomaly. These graphs are calculated in Ref. [5] for arbitrary m_t with $m_u = m_d = m_s = m_c = m_b = 0$. For $m_t \gg \sqrt{s}$ the result has a simple expansion [5],

$$
\mathcal{L} = \mathcal{R}_b^i(0) \left\{ \log \frac{m_t^2}{s} - 3.083 + 0.0865 \frac{s}{m_t^2} + 0.0132 \left(\frac{s}{m_t^2} \right)^2 + \cdots \right\}.
\tag{8}
$$

Corrections to Eq. (8) proportional to m_b^2/s are given in Ref. [6].

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FIG. 2. Graph for $\mathcal{L}$, Eq. (6).

In the order α_s^3 term, R_3, it suffices for practical purposes to simply set $m_L = 0$, and keep only terms that do not vanish as $m_H \to \infty$. The large m_H form of those graphs that are order α_s^3 corrections to $\mathcal{L}$ has been calculated in [3], and the rest of R_3 has been calculated in the $m_L = 0$, $m_H = \infty$ limit in [2]. These terms amount to about a 1% correction in the value of α_s extracted from R_Z.

Our aim is to calculate the functions $F(m^2/s)$ and $G(m^2/s)$. To begin, we notice that these functions are related. To see this, consider R_Z at orders α_s^0, α_s^1, and α_s^2, ignoring the contribution $\mathcal{L}$. Focus attention on the contributions from a particular choice $i = V$ or A

and from a particular light flavor f in the outside loop. For our present purposes, it suffices to take $m_f = 0$ and $\mu = \sqrt{s}$. Call the corresponding function δR_f^i:

$$
\begin{aligned}
\delta R_f^i = \mathcal{R}_f^i(0) \Bigg\{ 1 + \frac{\alpha_s(N, \sqrt{s})}{\pi} \\
+ \left(\frac{\alpha_s(N, \sqrt{s})}{\pi} \right)^2 \Big[1.985707 - 0.115295\, N + \\
+ \sum_{f' \leq N} F(m_{f'}^2/s) + \sum_{f' > N} G(m_{f'}^2/s) \Big] \Bigg\}.
\end{aligned}
\tag{9}
$$

We have made explicit the fact that the definition of the strong coupling α_s depends on the number of flavors, N, that are fully included in loops using the $\overline{\text{MS}}$ prescription. This number N is normally taken to be the number of light flavors N_L, as defined by the condition $m_{f'} < \sqrt{s}/2$. However, this is a convention that could be varied. What happens if we change conventions and consider the heaviest of the light quarks play the role of a heavy quark in Eq. (9)? Let the flavor index of this quark be $f' = q$. Then the explicit factor of N decreases by 1 and the value of the strong coupling changes,

$$
\frac{\alpha_s(N, \sqrt{s})}{\pi} = \frac{\alpha_s(N-1, \sqrt{s})}{\pi} - \frac{1}{6} \left(\frac{\alpha_s(N-1, \mu)}{\pi} \right)^2 \log \left(\frac{m_q^2}{s} \right) + O(\alpha_s^3).
\tag{10}
$$

Furthermore, we replace $F(m_q^2/s)$ by $G(m_q^2/s)$. However the contribution δR_f^i to the physical Z-width must remain the same up to corrections of order α_s^3. This requires that

$$
G\left(\frac{m_q^2}{s} \right) = F\left(\frac{m_q^2}{s} \right) - \frac{1}{6} \log \left(\frac{m_q^2}{s} \right) - 0.115295.
\tag{11}
$$

We will utilize this relation by calculating both F and G independently and verifying that Eq. (11) is satisfied. Notice that, from its definition, $F(m^2/s) \to 0$ when $m \to 0$, while the decoupling theorem guarantees that $G(m^2/s) \to 0$ when $m \to \infty$. For small m, it is F that appears in Eq. (6), while for large m it is G.

We now describe the calculation of $F(m^2/s)$. It suffices to consider the contribution from vector currents $j^\mu = \bar{q}_f \gamma^\mu q_f$, where f denotes one of the light flavors, the mass of which we take to be zero. The derivation for axial vector currents is similar, and the results for F and G are the same. We define the function $\Pi(q^2)$ in terms of the two-point correlation function of the two vector currents:

$$
i \int e^{iqx} < 0|Tj^\mu(x)j^\nu(0)|0 > d^4x = (q^\mu q^\nu - g^{\mu\nu}q^2)\Pi(q^2) .
\tag{12}
$$

On the right hand side of Eq. (12), one can separate the transverse tensor factor because of current conservation. The corresponding contribution to the hadronic decay width is determined by the discontinuity of $\Pi(q^2)$ across the positive real q^2-axis. To calculate $F(m^2/s)$ according to Eq. (9), we need to consider the contribution to $\Pi(q^2)$ from the graphs in Fig. 1, where the quark in the outer loop is massless and the quark in the inner loop has mass m. Call this contribution $\Pi_G(q^2, m)$. Then

$$\frac{1}{2\pi i}\left[\Pi_G(s+i\epsilon, m) - \Pi_G(s-i\epsilon, m)\right] = \frac{1}{12\pi^2}\left(\frac{\alpha_s}{\pi}\right)^2\left[-0.115295 + F(m^2/s)\right]. \qquad (13)$$

Here the normalization factor $1/(12\pi)$ is determined by the Born graph. As dictated by Eq. (9), we understand the graphs to include their renormalization subtractions in the $\overline{\text{MS}}$ scheme. In particular, we subtract the pole part of the heavy quark loop.

On the right hand side of Eq. (13), the value of the graphs for a massless quark is represented by the constant term. The mass dependence is contained in $F(m^2/s)$, where we have defined $F(0) = 0$. It is useful to extract $F(m^2/s)$, by subtracting the $m = 0$ graphs:

$$\frac{1}{12\pi^2}\left(\frac{\alpha_s}{\pi}\right)^2 F(m^2/s) = \qquad\qquad\qquad\qquad\qquad (14)$$
$$\frac{1}{2\pi i}\left[\Pi_G(s+i\epsilon, m) - \Pi_G(s-i\epsilon, m)\right] - \frac{1}{2\pi i}\left[\Pi_G(s+i\epsilon, 0) - \Pi_G(s-i\epsilon, 0)\right].$$

The advantage is that if we take the difference inside the integrals then the loop integrations are convergent. We write the three-loop diagrams of Fig. 1, subtracted at zero mass, in the form of two loop effective diagrams as shown in Fig. 3. The double wavy line is the effective gluon propagator. It is constructed from the gluon propagator with a quark loop insertion,

$$\frac{i}{k^2 + i\epsilon}\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2}\right)\mathcal{P}(k^2, m^2), \qquad\qquad (15)$$

by replacing $\mathcal{P}(k^2, m^2)$ by $\mathcal{P}(k^2, m^2) - \mathcal{P}(k^2, 0)$. We use

$$\mathcal{P}(k^2, m^2) - \mathcal{P}(k^2, 0) = \frac{2m^2\alpha_s}{\pi}\int_0^1 d\alpha \int_0^1 d\beta \, \frac{1}{k^2 - M(\alpha, \beta)^2 + i\epsilon}, \qquad (16)$$

where $M(\alpha, \beta)^2 = m^2\beta/[\alpha(1-\alpha)]$. We write the diagrams of Fig. 3 as integrals over Feynman parameters and integrate analytically over as many of the Feynman parameters as possible. This involves a certain amount of computer algebra, for which we use FORM [11]. The structure of the result is simple enough that we can take the discontinuity indicated in Eq. (14) analytically. Then we perform the remaining Feynman parameter integrals numerically by Monte Carlo integration. (We use VEGAS [12].)

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FIG. 3. The two loop effective diagrams.

Given $F(m^2/s)$, one can calculate $G(m^2/s)$ from Eq. (11). However, this procedure loses numerical accuracy for large m^2/s, where the small number $G(m^2/s)$ is obtained from Eq. (11) as the difference of two large numbers. A better procedure it to calculate $G(m^2/s)$ directly. An added advantage of calculating $G(m^2/s)$ directly is that Eq. (11) then provides a check on both calculations.

To calculate $G(m^2/s)$ directly, we use the Collins-Wilczek-Zee (CWZ) renormalization prescription [13]. For the sake of definiteness, let us say that the heavy quark with mass m is the top quark. The CWZ prescription is to renormalize the ultraviolet divergences for all subgraphs that do *not* contain top quark lines according to the $\overline{\text{MS}}$ prescription. Subgraphs

that *do* contain top quark lines are renormalized by subtraction at zero momentum. Then the value of a renormalized graph with external particles consisting of light quarks and gluons and with external momenta that are small compared to m is the $\overline{\text{MS}}$ value if the graph has no top quark loops, and zero in the large m limit if the graph does have top quark loops. Thus α_s in this scheme is to be identified with the five-flavor $\overline{\text{MS}}$ version of α_s. Now in Eq. (6) or Eq. (9), the function $G(m^2/s)$ is defined to be the function that reflects the effect of top quark loops on R_Z at the three loop order of perturbation theory, but this means perturbation theory in the five-flavor $\overline{\text{MS}}$ version of α_s.

Thus to calculate $G(m^2/s)$ we have only to calculate the graphs of Fig. 1 in the CWZ renormalization scheme. For these graphs, all of the divergent subgraphs contain the heavy quark loop, so all of the subtractions are at zero momentum. Since no $\overline{\text{MS}}$ subtractions are needed, it suffices to perform the entire calculation in exactly four dimensions. From these graphs, we construct $G(m^2/s)$ using

$$\frac{1}{12\pi^2} \left(\frac{\alpha_s}{\pi}\right)^2 G(m^2/s) = \frac{1}{2\pi i} \left[\Pi_G^{\text{CWZ}}(s + i\epsilon, m) - \Pi_G^{\text{CWZ}}(s - i\epsilon, m)\right]. \tag{17}$$

which is the analogue of Eq. (14).

As with the calculation of $F(m^2/s)$, we can calculate the three loop graphs as two loop graphs with an effective gluon propagator, as shown in Fig. 3. In this case the effective gluon propagator is given by Eq. (15) with $\mathcal{P}(k^2, m^2)$ replaced by $\mathcal{P}(k^2, m^2) - \mathcal{P}(0, m^2)$, reflecting the CWZ zero-momentum subtraction. We represent this as

$$\mathcal{P}(k^2, m^2) - \mathcal{P}(0, m^2) = -\frac{\alpha_s}{\pi} \int_0^1 d\alpha \, \frac{\alpha^2 \left(1 - \alpha^2/3\right)}{1 - \alpha^2} \frac{k^2}{k^2 - M(\alpha)^2 + i\epsilon}, \tag{18}$$

where $M(\alpha)^2 = 4m^2/(1 - \alpha^2)$. Some zero-momentum subtractions are called for in the effective graphs of Fig. 3. We need not worry about the overall subtraction, since it is a constant, independent of q^2, and hence its discontinuity across the positive q^2 axis vanishes. There are, however, one loop subgraphs in Fig. 3 that are divergent by power counting and should be subtracted at zero momentum. In the second and third graphs, there are divergent quark self-energy subgraphs of the form $A(k^2) \, k \cdot \gamma$. According to the CWZ prescription, these are replaced by $[A(k^2) - A(0)] \, k \cdot \gamma$. In the first graph of Fig. 3, there is a surprise: the counter-terms vanish when calculated in exactly four dimensions because they have a factor of zero arising from the numerator algebra. Thus the integral corresponding to this diagram is finite.

As with the calculation of $F(m^2/s)$, we write the diagrams of Fig. 3 as integrals over Feynman parameters. We take the discontinuity indicated in Eq. (17) numerically, by choosing a small but finite value for ϵ. Then we perform the Feynman parameter integrals numerically by Monte Carlo integration. (We checked that the result does not depend on ϵ to within our integration errors.)

In Table I we present our numerical results for $F(m^2/s)$ and $G(m^2/s)$ for various values of quark mass m at $\sqrt{s} = M_Z = 91.188 GeV$. The integration error is $\lesssim 5$ in the last digit. Using these data, we verify (to within the integration errors) Eq. (11), which relates F and G.

TABLE I. Calculated values of $F(m^2/s)$ and $G(m^2/s)$.

m(GeV)	$F(m^2/M_Z^2)$	$G(m^2/M_Z^2)$	m(GeV)	$F(m^2/M_Z^2)$	$G(m^2/M_Z^2)$
$m_c = 1.3$	0.00000031		100	0.20160	0.0553
$m_b = 4.72$	0.0000392	0.869	110	0.22572	0.0478
10	0.000563	0.621	120	0.24867	0.0418
15	0.002199	0.487	130	0.2703	0.0367
20	0.00556	0.395	140	0.2909	0.0325
30	0.01854	0.2731	150	0.31056	0.0293
40	0.03933	0.1983	170	0.3467	0.0238
50	0.06523	0.1496	180	0.3637	0.0216
60	0.09335	0.1173	200	0.3957	0.01825
70	0.12174	0.0943	250	0.4639	0.01256
80	0.14953	0.0772	600	0.7456	0.00277
M_Z	0.1793	0.0635	1000		0.001118

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FIG. 4. Calculated values of $F(m^2/s)$ and $G(m^2/s)$ versus m for $\sqrt{s} = 91.188$ GeV.

These results are plotted in Fig. 4. For small m^2/s, $F(m^2/s)$ is the physically relevant function in Eq. (6). We see from the table or the figure that F vanishes in this limit. A reasonable (2%) approximation for $0.05 \lesssim m/\sqrt{s} \lesssim 0.3$ is

$$F(m^2/s) \approx \frac{m^4}{s^2} \times \left\{ -0.474894 + \log(s/m^2) + \frac{m}{\sqrt{s}} \left[-0.5324 - 0.0185 \log(s/m^2) \right] \right\}. \quad (19)$$

The first two terms here are taken from the recent evaluation of Chetyrkin [14]. We confirm these coefficients within the limits of our numerical calculation. The next terms result from a numerical fit to our data.

For large m^2/s, $G(m^2/s)$ is the physically relevant function. We see that G vanishes in this limit. That is, the heavy quark decouples [10]. A reasonable (1%) approximation for $m/\sqrt{s} \gtrsim 1$ is

$$G(m^2/s) \approx \frac{s}{m^2} \times \left\{ \frac{44}{675} + \frac{2}{135} \log(m^2/s) - \frac{\sqrt{s}}{m} \left[0.001226 + 0.001129 \log(m^2/s) \right] \right\}. \quad (20)$$

The first two terms here are taken from a recent evaluation of Chetyrkin [7]. We confirm these coefficients within the limits of our numerical calculation. The remaining terms result from a numerical fit to our data. The fact that these terms are small indicates that the large mass expansion works well. The present calculation thus serves as a direct test confirming the usefulness of the methods of Euclidean asymptotic expansions of Feynman integrals (see [15] and references therein).

In summary, this calculation completes in a certain sense the evaluation of the top quark mass dependence of $O(\alpha_s^2)$ corrections to the hadronic decay width of Z-boson. We have found that quark mass dependence can be evaluated rather simply, without depending on expansions in m^2/s or s/m^2, in diagrams of a fairly high order. We expect that the results may be useful for the analysis of $\Gamma(\tau^- \to \text{hadrons})$.

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